

Research article

# MATHEMATICAL MODEL TO PREDICT PATHOGEN DEPOSITION INFLUENCED BY DEGREE OF SATURATION IN ORGANIC AND LATERITIC SOIL IN WASTE DUMPS SITE OF PORT HARCOURT METROPOLIS

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## Abstract

The deposition of pathogen in organic and lateritic soil at high percentage was expressed through risk assessment, the study was to determine the level of risk from waste dump site in Port Harcourt metropolis high rate of pollution source from pathogen in some selected site through risk assessment were carried out, the study express the rate of predominant deposition of pathogen in organic and lateritic soil, the investigation developed the rate of accumulation but could not develop thorough prevention of managing the deposition of this regenerated contaminants in the study area, base on this factors, development of mathematical model were found suitable to express the rate of accumulation and predict further migration that will pollute shallow aquifer in the study area. The deltaic nature of the formation are influenced by climatic conditions under the pressured of high rain intensities, these were considered in developing the expressed governing equation, the derived model will definitely monitor the deposition of pathogen in the study location, it will also express further prediction on the migration of the contaminant in organic and lateritic soil in waste dump site of port Harcourt metropolis.

**Keywords:** mathematical model, pathogen deposition degree of saturation and dump site

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## 1. Introduction

Over the last few decades deterioration of both the quality and quantity of groundwater has become a global phenomenon, which will further intensify the demand for drinking Water increases (World Bank report, 2006). Numerous severe cases of groundwater contamination with reference to storm water infiltration have been documented worldwide (Lopes and Bender, 1998; Fischer, 2003, Zubair, 2007). Few studies have also been documented nationally on groundwater with reference to major ions, trace elements and bacteriology (Rahman et. al., 1997; Zubair, 1998, Zubair, 2007). However literature is silent on the impact of storm water infiltration into groundwater. In recent years attention on the increasing ionic concentration of traces metals in groundwater as result of storm water infiltration has been studied by various workers (Ku et. al., 1992; Appleyard, 1993; Wild, 1994; Hathhorn and Yonge, 1995; Pitt, 1996; Lopes and Bander, 1998; Fisher, 2003, Eluozo and Nwaoburu 2013). These have been attributed to human interference, proliferation of industries and recent agriculture practices in urban areas where storm water flow recharges the aquifer system and thus degrading the water quality. It is often difficult to determine the exact source of major ions pollutants (Ford, 1990), because there are many potential sources of groundwater contamination including urban storm water runoff. Storm water infiltration in urban areas is cause of concern with regard to the risk of groundwater pollution (Mikkelsen et. al., 1994, Zubair, 2007). Storm water infiltration has been shown to affect groundwater quality and quantity (Pitt, 1996; Fisher, 2003). Contaminants present in urban storm water include volatile organic compound, pesticides, nutrients, and trace elements (Fisher, 2003). This can originate at the land surface or in the atmosphere (Lopes and Bender, 1998). Some constituents either volatilize during storage or sorbs to the particulate matter (Mikkelsen et. al., 1996, Eluozo and Nwaoburu 2013) and are not transported to the water table; however, are more persistent, and may threaten groundwater quality. Malmquist & Hard (1981) studied the impacts from sub surface infiltration at three sites in Sweden and concluded that storm water infiltration affects the groundwater quality to a small extent. In a vast majority of developing countries, fast growing populations combined with poor living conditions in rural areas have forced many people to migrate to cities in search of better living conditions. This has led to a dramatic expansion of most of the major cities throughout developing countries, mainly via the uncontrolled growth of slums or squatter settlements on their fringes (Khadam, 1988; Pathak, 1994; Wang et. al., 2003). Nitrogen is one of the most abundant elements in the Earth's biosphere and one of the six elements (C, H, O, N, P, and S) that are the major constituents of living tissue. Nitrogen gas (N<sub>2</sub>) comprises approximately 78% of the Earth's atmosphere, but this is largely unavailable as a nitrogen source for most living organisms. Consequently, nitrogen availability in all ecosystems is largely dependent on inputs of biologically available nitrogen from external sources or internal cycling of nitrogenous compounds into biologically available forms. Nitrogen often limits biological production in estuaries, oceans, and many terrestrial systems (Schlesinger 1997), and can be limiting in lakes (White et al. 1985, Dodds et al. 1993), streams (Grimm and Fisher 1986, Hill and Knight 1988), and wetlands (Shaver et al. 1986). However, excess nitrogen can have detrimental effects. For example, excess nitrogen or nitrogen saturation (nitrogen losses approaching nitrogen inputs) can lead to

increased losses of nutrient cations and increased soil and water acidity in forest ecosystems (Vitousek et al. 1997). In aquatic habitats, excess nitrogen can lead eutrophication or levels of ammonia (NH<sub>3</sub>), nitrite (NO<sub>2</sub><sup>-</sup>), and nitrate (NO<sub>3</sub><sup>-</sup>) toxic to humans, livestock, and wildlife (Cairns et al. 1990, Carpenter et al. 1998, Marco et al. 1999, Soupir, et al 2006). Land application of waste from confined animal production facilities is an effective method of disposing of animal waste while supplying nutrients to crops and pastureland. However, it has been well-documented that runoff from agricultural livestock and poultry litter applied areas is a source of fecal contamination in water (Crowther et al., 2002; Edwards et al., 1994, 2000; Gerba and Smith, 2005; Tian et al., 2002). The EPA's National Water Quality Inventory report (USEPA, 2000) identified bacteria as the leading cause of impairments in rivers and streams in the United States and agricultural practices were identified as the leading source of all bacterial impairments (Eluozo and Nwaoburu 2013).

## 2. Theoretical background

The depositions of microbes are very common in any waste dump site, although there several organisms that deposit in where there are biological waste generations, the accumulations of pathogen in the environments were investigated in the dump site environment in port Harcourt metropolis, it was confirmed from risk assessment that pathogen are predominant within the organic and lateritic soil in the dump site environments, other microorganism are detected from the investigation through risk assessment carried out in the study area, the purpose was to monitor the level of risk generated from biological waste dumped without treatment, accumulation of pathogen predominant were found to deposit at high percentage in organic and lateritic soil in port Harcourt environ dump site, climatic condition were confirm to influence the accumulation or migration of pathogen in organic and lateritic soil formation, the deposition of pathogen accumulating in organic and lateritic soil is a serious concern in different dimension, subject to this condition in soil, the influential parameters in the system of pathogen are found to develop high degree of saturation, due the level climatic condition, high rain intensities are found to develop high degree in the study area, the deltaic nature of the formation establish high degree of saturation of the organic and lateritic soil, although the permeation of organic and lateritic are very low, these develop high percent of pathogen in the stated formation, the deposition of pathogen in organic and lateritic soil are base in constant regeneration of the waste, the accumulation of the microbes continue to deposit in the two formation through constant regeneration of the waste in the dump site, the study is imperative because the formation under the influenced by degree of, this increase high rain intensities and such condition continue to increase the deposition of pathogens in the soil, the fluid flow movement will migrate pathogens to short fresh water aquifer that may deposit within the environ of the waste dump site, the study area is deltaic in nature and the deposition of shallow aquifer are confirm through hydrogeological studies to be predominant, the influences from degree of saturation in the study location are not peculiar,

therefore such development in the stratification of the formation are not peculiar in the deltaic environments, this implies that the deposition of pathogen are generated at this phase through influences from high degree of saturation, the tendency of these deposition to migrate through flow paths in the soil deposition and pollute short fresh water aquifers may definitely be at ease in study location. The study is to monitor the deposition of pathogen in organic and lateritic soil and its accumulation under the influences of high degree of saturation from climatic influences, the migration of pathogen base on the level of soil deposition are confirm to migrate pathogen through the influences of high rain intensities in the system. Such expression reflect the accumulation and migration of pathogen through fluid flow path in soil

### 3. Governing equation

$$\theta_w V \frac{\partial C}{\partial t} = Om \frac{\rho_b}{\rho_w} V \frac{\partial C^2}{\partial x^2} \dots\dots\dots (1)$$

Examining the study location on the waste generation has been found to influence lots of pollution which has cause lot ill health within the settler in the environ, the study focuses on the predominant deposition of pathogen in the waste dump site since risk assessment has investigate the predominant of pathogen in the dump site, therefore mathematical approach were found appropriate to develop a model that can monitor these predominant microbes in organic and lateritic soil, the governing equation were develop base on these stated conditions in the system

#### Nomenclature.

$\phi W_v$  = mass of transport

$Om$  = degree of saturation

$V$  = void ratio

$P_{b/PW}$  = bulk density

$V$  = velocity

$C$  = pathogen

Substituting solution  $C = XT$  into (1), we have

$$\theta_w V XT^1 = \theta_m \frac{\rho_b}{\rho_w} V X^{11} T \dots\dots\dots (2)$$

$$\theta_w V \frac{T^1}{T} = - \theta_m \frac{\rho_b}{\rho_w} V \frac{X^{11}}{X} \dots\dots\dots (3)$$

$$\theta_w V \frac{T^1}{T} - \theta m \frac{\rho_b}{\rho_w} V \left[ \frac{X^{11}}{X} \right] \dots\dots\dots (4)$$

$$\theta_w V \frac{T^1}{T} - \frac{X^{11}}{X} \dots\dots\dots (5)$$

Considering when  $\ln X \rightarrow 0$

$$\theta_w V T^1 = \theta m \frac{\rho_b}{\rho_w} V \frac{X^{11}}{X} - T = \lambda^2 \dots\dots\dots (6)$$

$$\theta_w V \frac{T^1}{T} = \lambda^2 \dots\dots\dots (7)$$

$$\frac{X^{11}}{X} = \lambda^2 \dots\dots\dots (8)$$

$$\theta m \frac{\rho_b}{\rho_w} V = \lambda^2 \dots\dots\dots (9)$$

This implies that equation (10) can be expressed as:

$$\theta m \frac{\rho_b}{\rho_w} V \frac{X^{11}}{X} = \lambda^2 \dots\dots\dots (10)$$

$$\theta m \frac{\rho_b}{\rho_w} V \frac{X^2}{X} = \lambda^2 \dots\dots\dots (11)$$

$$\theta_w V \frac{d^2 y}{dx^2} = \lambda^2 \dots\dots\dots (12)$$

$$\theta_w \frac{\rho_b}{\rho_w} V \frac{d^2 y}{dx} = \lambda^2 \dots\dots\dots (13)$$

$$\theta_w \frac{d^2 y}{dx^2} = \lambda^2 \dots\dots\dots (14)$$

$$\frac{d^2 y}{dx} = \frac{\lambda^2}{\theta_w V} \dots\dots\dots (15)$$

$$d^2 y = \left[ \frac{\lambda^2}{\theta_w V} \right] dx^2 \quad \dots\dots\dots (16)$$

$$\int d^2 y = \int \frac{\lambda^2}{\theta_w V} dx^2 \quad \dots\dots\dots (17)$$

$$dy = \frac{\lambda^2}{\theta_w V} x dx \quad \dots\dots\dots (18)$$

$$\int dy = \int \frac{\lambda^2}{\theta_w V} X dx + C_1 \quad \dots\dots\dots (19)$$

$$y = \frac{\lambda^2}{\theta_w V} + C_1 + C_2 \quad \dots\dots\dots (20)$$

$$y = 0 \quad \dots\dots\dots (21)$$

$$\Rightarrow \frac{\lambda^2}{\theta_w V} X^2 C_{1x} + C_2 = 0 \quad \dots\dots\dots (22)$$

Several derived mathematical methods usually come into mind in transport of any substance that will move with fluid, derived model solution use separation of variables, this is to ensure that all the parameter express their function in other to confirm some of the influential condition that establish the deposition, accumulation of pathogen in the organic and lateritic soil are expressed in the stage of the derived solution, mathematical approach were found necessary in this direction. Several conditions are considered on the processes of expressing the derived solution by applying these concepts. the application of separation of variable was in line at these condition of monitoring the deposition of pathogen in organic and lateritic soil., but further conditions were express in the derived solution as it is detailed in the governing equation therefore application quadratic expression was suitable at the stage of the derived solution in the study, application of quadratic function is to ensure that all the parameters express their various relation to determine various influences that develop high rate of accumulation through the expression of integration by interaction of both parameters in the system.

Applying quadratic expression, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \dots\dots\dots (23)$$

Where  $a = \frac{\lambda^2}{\theta w V}$ ,  $b = C_1$  and  $c = C_2$

$$X = \frac{-(C_1) \pm \sqrt{(C_1)^2 - 4 \left( \frac{\lambda^2}{\theta w V} \right) C_2}}{2 \frac{\lambda^2}{\theta w V}} \dots\dots\dots (24)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_2 \frac{\lambda^2}{\theta w V}}}{2 \frac{\lambda^2}{\theta w V}} \dots\dots\dots (25)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_2 \frac{\lambda^2}{\theta w V}}}{2\theta w V} \dots\dots\dots (26)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_2 \frac{\lambda^2}{\theta w V}}}{2\theta w V} \dots\dots\dots (27)$$

$$X = \frac{-C_1 - \sqrt{C_1^2 - 4C_2 \frac{\lambda^2}{\theta w V}}}{2 \frac{\lambda^2}{\theta w V}} \dots\dots\dots (28)$$

Substituting equation (20) to the following condition and initial values condition.

$$t = 0, C = 0 \dots\dots\dots (29)$$

Therefore,  $X_{(x)} = C_1 e^{-x} - e^{-mx} + C_2 M^{em2x} \dots\dots\dots (30)$

$$C_1 \text{Cos} M_{1x} + C_2 \text{Sin} M_{2x} \dots\dots\dots (31)$$

$$y = \frac{\lambda^2}{\theta w V} + C_1 + C_2 \dots\dots\dots (32)$$

$$C(x,t) = \left[ C_1 \cos M_1 \frac{\lambda^2}{\theta_w V} x + C_2 \sin M_2 \frac{\lambda^2}{\theta_w V} x \right] \dots\dots\dots (33)$$

But if  $x = \frac{v}{t}$

Therefore, equation (33) can be expressed as:

$$C(x,t) = \left[ C_1 \cos M_1 \frac{\lambda^2}{\theta_w V} \frac{v}{t} + C_2 \sin M_2 \frac{\lambda^2}{\theta_w V} \frac{v}{t} \right] \dots\dots\dots (34)$$

The expression in [34] is the developed model that will monitor pathogenic deposition in organic and lateritic soil; the developed mathematical model was found suitable due to high rate of predominant pathogenic origin the study area, the study was to monitor pathogen at organic and lateritic soil level and predict the migration of the microbes under the influences of high degree of saturation through climatic condition. These conditions were thoroughly considered in the study area to predict the time of further migration influenced by constant regeneration and high degree of saturation in the formation

#### 4. Conclusion

The investigation from risk assessment carried has developed a fruitful studies that will definitely improve health status of the people in the environs, pathogen deposition were confirm to be predominant from risk assessment carried in waste dump site of port Harcourt metropolis, such condition has lots of environmental negative impact in human settlement ,but the focus of these study centred on the predominant deposition of pathogen that need to be engineered out of the soil or prevented for further pollution mostly our shallow aquifers, several methods can be applied to prevent this pollution in the study area, but will not perfectly determine the sources and the rate of migration within the sample zone, therefore the establishments of mathematical modeling approach were found necessary to mathematical model the deposition of pathogen in organic and lateritic soil, the derived mathematical model will serve as a base line for further investigation to monitor or predict further migration that will pollute shallow aquifer in the study area.

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